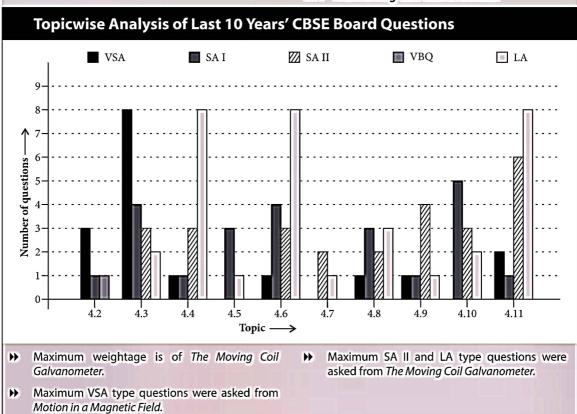


Moving Charges and Magnetism

- 4.2 Magnetic Force
- 4.3 Motion in a Magnetic Field
- 4.4 Motion in Combined Electric and Magnetic Fields
- 4.5 Magnetic Field Due to a Current Element, Biot-Savart Law
- 4.6 Magnetic Field on the Axis of a Circular

Current Loop

- 4.7 Ampere's Circuital Law
- 4.8 The Solenoid and the Toroid
- 4.9 Force between Two Parallel Currents, the Ampere
- 4.10 Torque on Current Loop, Magnetic Dipole
- 4.11 The Moving Coil Galvanometer



QUICK RECAP

Magnetic field: It is a region or space around a magnet or current carrying conductor or a

moving charge, in which its magnetic effect can be felt.

► Magnetic field is a vector quantity and its dimensional formula is [ML⁰T⁻²A⁻¹]. The S.I. unit of magnetic field is tesla (T) or weber/metre² (Wb/m²). The CGS unit of magnetic field is gauss (G).

 $1 \text{ tesla} = 10^4 \text{ gauss}$

- ► The sources of magnetic fields are :
 - A current carrying conductor
 - Changing electric field
 - Moving charged particle
 - Permanent magnet and electromagnet etc.
- ► Conventionally the direction of the field perpendicular to the plane of the paper is represented by ⊗ if into the page, and by ⊙ if out of the page.

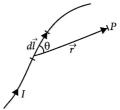
Force on a charged particle in a uniform magnetic field:

When a charged particle of charge q, moving with velocity \vec{v} is subjected to a uniform magnetic field \vec{B}_i , the force acting on it is

$$\vec{F} = q (\vec{v} \times \vec{B})$$
 or $F = qvB \sin \theta$

where θ is the angle between \vec{v} and \vec{B} .

- The direction of this force is perpendicular to the plane containing \vec{v} and \vec{B} .
- $\vec{F} = 0$ if $\vec{v} = 0$, *i.e.* a charge at rest does not experience any magnetic force.
- $\vec{F} = 0$ if $\theta = 0^{\circ}$ or 180° *i.e.*, the magnetic force vanishes if \vec{v} is either parallel or antiparallel to the direction of \vec{B} .
- Force will be maximum if $\theta = 90^{\circ}$, *i.e.*, if \vec{v} is perpendicular to \vec{B} , the magnetic force has a maximum value and is given by $F_{\text{max}} = qvB$.
- **D Lorentz force**: When a charged particle of charge q moving with velocity \vec{v} is subjected to an electric field \vec{E} and magnetic field \vec{B} , the total force acting on the particle is $\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$ This force is known as Lorentz force.
- Biot Savart's law:
 According to this law, the magnetic field at a point P due to a current element of length $d\vec{l}$, carrying current I, at a distance r from the element is

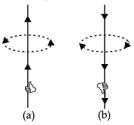


$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2} \quad \text{or} \quad d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

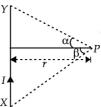
where θ is the angle between $d\vec{l}$ and \vec{r} and μ_0 is the permeability of free space.

$$\begin{split} &\mu_0 = 4\pi \times 10^{-7} \; T \; m \; A^{-1} = 4\pi \times 10^{-7} \; Wb \; A^{-1} \; m^{-1} \\ ∨ \quad \frac{\mu_0}{4\pi} = 10^{-7} \; henry/m. \end{split}$$

- The dimensional formula of μ_0 is $[MLT^{-2}A^{-2}]$. The direction of $d\vec{B}$ is that of $d\vec{l} \times \vec{r}$, which is perpendicular to the plane containing $d\vec{l}$ and \vec{r} , and is directed as given by right hand screw rule.
- The direction of magnetic field due to a straight current carrying wire is given by right hand thumb rule. According to this rule, if you grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers curling around the wire give the direction of the magnetic field lines.



Magnetic field due to a straight current carrying wire



► The magnetic field *B* at a point *P* due to a straight wire of finite length carrying current *I* at a perpendicular distance *r* is

$$B = \frac{\mu_0 I}{4\pi r} [\sin \alpha + \sin \beta]$$

Special cases

- If the wire is of infinite length and the point *P* lies near the centre of the straight wire, then $\alpha = \beta = 90^{\circ}$

$$\therefore B = \frac{\mu_0 2I}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$

- If the wire is of infinite length and the point P lies near one end, then $\alpha = \frac{\pi}{2}$, $\beta = 0^{\circ}$

$$\therefore B = \frac{\mu_0 I}{4\pi r}$$

Magnetic field at the centre of a current carrying circular loop: The magnetic field at the centre of a circular coil of radius a carrying current I is



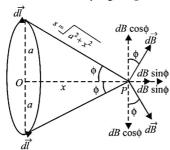
$$B = \frac{\mu_0}{4\pi} \frac{2\pi I}{a} = \frac{\mu_0 I}{2a}$$

If the circular loop consists of *N* turns, then

$$B = \frac{\mu_0}{4\pi} \frac{2\pi NI}{a} = \frac{\mu_0 NI}{2a}$$

The direction of magnetic field at the centre of a circular coil carrying current is given by right hand thumb rule.

Magnetic field at a point on the axis of a circular current carrying loop



The magnetic field at a point on the axis of the circular current carrying coil is

$$B = \frac{\mu_0}{4\pi} \frac{2\pi N I a^2}{(a^2 + x^2)^{3/2}}$$

where a is the radius of coil, x is the distance of the point on the axis from the centre of the coil, *N* is the number of turns in the coil.

Special cases:

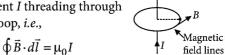
- If the point lies at the centre of the coil, i.e.

$$B = \frac{\mu_0}{4\pi} \frac{2\pi N I a^2}{(a^2)^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N I}{a} = \frac{\mu_0 N I}{2a}$$

$$B = \frac{\mu_0}{4\pi} \frac{2\pi N I a^2}{x^3} = \frac{\mu_0}{4\pi} \frac{2NIA}{x^3} = \frac{\mu_0}{4\pi} \frac{2M}{x^3}$$

where, NIA = M = magnetic dipole moment of current loop, A = cross sectional area of loop.

Ampere's circuital law: The line integral of the magnetic field B around any closed loop is equal to μ_0 times the total current I threading through the loop, i.e.,



Magnitude of magnetic field due to a straight wire using Ampere's circuital law

$$B = \frac{\mu_0 I}{2\pi r}$$

Maxwell introduced the concept of displacement

Displacement current,
$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

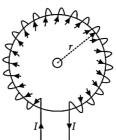
Displacement current flows in the space due to a variation in electric field.

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 (I_C + I_D)$$

Magnetic field due to a straight solenoid



- At any point inside the solenoid, $B = \mu_0 nI$ where, n = number of turns per unit length.
- At the ends of the solenoid, $B = \frac{1}{2}\mu_0 nI$
- Magnetic field due to toroidal solenoid



Inside the toroidal solenoid,

$$B = \mu_0 nI$$
, where $n = \frac{N}{2\pi r}$,

N = total number of turns

- In the open space, interior or exterior of toroidal solenoid, the magnetic field is zero.
- Motion in a magnetic field: When a charged particle of charge q and mass m moves with velocity \vec{v} in a uniform magnetic field \vec{B} , the force acting on it is $F = qvB \sin\theta$. The following two cases arise:

Case I : When the charged particle is moving perpendicular to the field *i.e.* $\theta = 90^{\circ}$. In this case path is circular.

Radius of circular path is

$$R = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{qB}$$

and time period of revolution is

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

► Case II : When the charged particle is moving at an angle θ to the field (other than 0°, 90° or 180°). In this case, path is helical.

Due to component of v, perpendicular to \vec{B} , *i.e.*, $v_{\perp} = v \sin \theta$, the particle describes a circular path of radius R, such that

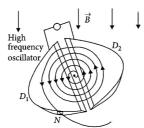
$$\frac{mv_{\perp}^2}{R} = qv_{\perp}B \quad \text{or} \quad R = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

Time period of revolution is

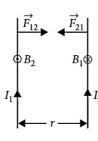
$$T = \frac{2\pi R}{v\sin\theta} = \frac{2\pi m}{qB}$$

- Cyclotron: The cyclotron is a machine to accelerate charged particles or ions to high energies. It was invented by E.O. Lawrence and M.S. Livingston in 1934.
- Cyclorton frequency $v = \frac{qB}{2\pi m}$
- ▶ The kinetic energy of the ions

$$\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$



Force between two parallel current carrying conductors: Two parallel conductors carrying currents in the same direction attract each other while those carrying currents in the opposite direction repel each other.



When two parallel conductors separated by a distance r carry currents I_1 and I_2 , the magnetic field of one will exert a force on the other. The

force per unit length on either conductor is $f = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{r}$

- The force of attraction or repulsion acting on each conductor of length l due to currents in two parallel conductors is $F = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{r} l$.
- When two charges q_1 and q_2 respectively moving with velocities v_1 and v_2 are at a distance r apart, then the force acting between them is

$$F = \frac{\mu_0}{4\pi} \frac{q_1 q_2 v_1 v_2}{r^2}$$

Torque on a current carrying coil placed in a uniform magnetic field: When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but different parts of the coil experience forces in different directions. Due to it, the coil may experience a torque or couple.

When a coil of area *A* having *N* turns, carrying current *I* is placed in a uniform magnetic field *B*, it will experience torque which is given by

$$\tau = NIAB\sin\theta = MB\sin\theta$$

where magnetic moment M = NIA and θ is the angle between the direction of magnetic field and normal to the plane of the coil.

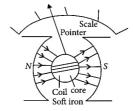
If the plane of the coil is perpendicular to the direction of magnetic field *i.e.* $\theta = 0^{\circ}$, then

$$\tau = 0$$
 (minimum)

If the plane of the coil is parallel to the direction of magnetic field *i.e.* $\theta = 90^{\circ}$, then

 $\tau = NIAB$ (maximum)

Moving coil galvanometer



It is an instrument used for the detection and measurement of small currents. It works on the principle that when a current carrying coil is placed in a magnetic field, it experiences a torque. In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection (θ).

$$I \propto \theta$$
 or, $I = G\theta$

where
$$G = \frac{k}{NAB} = \text{galvanometer constant}$$

A = area of a coil, N = number of turns in the coil, B = strength of magnetic field, k = torsional constant of the spring *i.e.* restoring torque per unit twist.

► Current sensitivity: It is defined as the deflection produced in the galvanometer, when unit current flows through it.

$$I_s = \frac{\theta}{I} = \frac{NAB}{k}.$$

The unit of current sensitivity is rad A^{-1} or div A^{-1} .

► Voltage sensitivity: It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

$$V_{s} = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{kR}$$

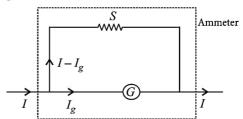
The unit of voltage sensitivity is rad V^{-1} or div V^{-1} .

$$V_{s} = \frac{1}{R} I_{s}$$

► Conversion of galvanometer into an ammeter: A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer, whose value is given by

$$S = \left(\frac{I_g}{I - I_g}\right)G$$

where I_g is the current for full scale deflection of galvanometer, I is the current to be measured by the galvanometer and G is the resistance of galvanometer.

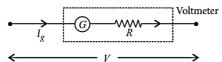


In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is S = G/(n-1).

Conversion of galvanometer into voltmeter: A galvanometer can be converted into voltmeter of given range by connecting a suitable resistance R in series with the galvanometer, whose value is given by

$$R = \frac{V}{I_g} - G$$

where V is the voltage to be measured, I_g is the current for full scale deflection of galvanometer and G is the resistance of galvanometer.



Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which potential difference is to be measured. An ideal voltmeter has infinite resistance.

In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is R = (n-1)G.

Previous Years' CBSE Board Questions

4.2 Magnetic Force

VSA (1 mark)

1. Write the expression, in a vector form, for the Lorentz magnetic force \vec{F} due to a charge moving with velocity \vec{v} in a magnetic field \vec{B} . What is the direction of the magnetic force?

(Delhi 2014)

2. Define one tesla using the expression for the magnetic force acting on a particle of charge 'q' moving with velocity \vec{v} in a magnetic field \vec{B} .

(Foreign 2014)

3. Use the expression : $\vec{F} = q \ (\vec{v} \times \vec{B})$, to define the S.I. unit of magnetic field. (AI 2010C)

SAI (2 marks)

4. Write the relation for the force F acting on a charge carrier q moving with a velocity \vec{v} through a magnetic field \vec{B} in vector notation. Using this relation, deduce the conditions under which this force will be (i) maximum (ii) minimum.

(Delhi 2007)

VBQ (4 marks)

- 5. Kamal's uncle was advised by his doctor to undergo an MRI scan test of his chest and gave him an estimate of the cost. Not knowing much about the significance of this test and finding it to be too expensive he first hesitated. When Kamal learnt about this, he decided to take help of his family, friends and neighbours and arranged for the cost. He convinced his uncle to undergo this test so as to enable the doctor to diagnose the disease, he got the test done and resulting information greatly helped the doctor to give him proper treatment.
 - (a) What, according to you, are the values displayed by Kamal?
 - (b) Assuming that the MRI scan test involved a magnetic field of 0.1 T, find the maximum and minimum values of the force that this

field could exert on a proton moving with a speed of 10⁴ m s⁻¹. State the condition under which the force can be minimum.

(Foreign 2013)

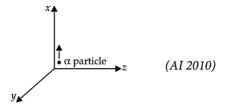
4.3 Motion in a Magnetic Field

VSA (1 mark)

- 6. A particle of mass 'm' and charge 'q' moving with velocity 'v' enters the region of uniform magnetic field at right angle to the direction of its motion. How does its kinetic energy get affected? (Delhi 2015C)
- 7. A long straight wire carries a steady current I along the positive y-axis in a coordinate system. A particle of charge + Q is moving with a velocity \vec{v} along the x-axis. In which direction will the particle experience a force? (Foreign 2013)
- 8. Depict the trajectory of a charged particle moving with velocity \vec{v} as it enters in a uniform magnetic field perpendicular to the direction of its motion. (AI 2012C)
- 9. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the circular paths described by them?

(Foreign 2011)

- 10. Two particles A and B of masses m and 2m have charges q and 2q respectively. Both these particles moving with velocities v_1 and v_2 respectively in the same direction enter the same magnetic field B acting normally to their direction of motion. If the two forces F_A and F_B acting on them are in the ratio of 1:2, find the ratio of their velocities. (*Delhi 2011C*)
- 11. A beam of α particles projected along + x-axis, experiences a force due to a magnetic field along the + y-axis. What is the direction of the magnetic field?



- 12. What is the direction of the force acting on a charged particle q, moving with a velocity \vec{v} in a uniform magnetic field \vec{B} ? (Delhi 2008)
- 13. An electron is moving along +ve *x*-axis in the presence of uniform magnetic field along +ve *y*-axis. What is the direction of the force acting on it?

 (AI 2007)

SAI (2 marks)

14. A particle of charge 'q' and mass 'm' is moving with velocity \vec{v} . It is subjected to a uniform magnetic field \vec{B} directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius.

(Foreign 2012)

- 15. A proton and a deuteron, each moving with velocity \vec{v} enter simultaneously in the region of magnetic field \vec{B} acting normal to the direction of velocity. Trace their trajectories establishing the relationship between the two. (*Delhi 2012C*)
- 16. Write the expression for Lorentz magnetic force on a particle of charge 'q' moving with velocity \vec{v} in a magnetic field \vec{B} . Show that no work is done by this force on the charged particle.

(AI 2011)

SA II (3 marks)

18. (a) Write the expression for the magnetic force acting on a charged particle moving with velocity v in the presence of magnetic field B.

- (b) A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown. Trace $\stackrel{\sim}{}$ \times \times \times \times \times \times \times their paths in the $\stackrel{\sim}{}$ \times \times \times \times \times \times \times their paths in the $\stackrel{\sim}{}$ \times \times \times \times \times \times \times \times field and justify your answer. (Delhi 2016)
- 19. A uniform magnetic field \vec{B} is set up along the positive *x*-axis. A particle of charge 'q' and mass 'm' moving with a velocity \vec{v} enters the field at the origin in *X-Y* plane such that it has velocity components both along and perpendicular to the magnetic field *B*. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation.

(AI 2015)

20. An electron moving horizontally with a velocity of 4×10^4 m/s enters a region of uniform magnetic field of 10^{-5} T acting vertically upward as shown in the figure. Draw its trajectory and find out the time it takes to come out of the region of magnetic field.

LA (5 marks)

21. A proton and a deuteron having equal momenta enter in a region of a uniform magnetic field at right angle to the direction of the field. Depict their trajectories in the field.

(2/5, Delhi 2013, AI 2010)

22. Deduce the expression for the frequency of revolution of a charged particle in a magnetic field and show that this is independent of the velocity or energy of the particle. (AI 2010C)

4.4 Motion in Combined Electric and Magnetic Fields

VSA (1 mark)

23. Write the condition under which an electron will move undeflected in the presence of crossed electric and magnetic fields. (AI 2014C)

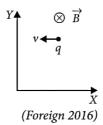
SA | (2 marks)

24. State the underlying principle of a cyclotron. Write briefly how this machine is used to accelerate charged particles to high energies.

(Delhi 2014)

SA II (3 marks)

- 25. (a) A point charge q moving with speed v enters a uniform magnetic field B that is acting into the plane of paper as shown. What is the path followed by the charge q and in which plane does it move?
 - (b) How does the path followed by the charge get affected if its velocity has a component parallel to \vec{B} ?
 - (c) If an electric field \vec{E} is also applied such that the particle continues moving along the original straight line path, what should be the magnitude and direction of the electric field \vec{E} ?



- **26.** Draw a schematic sketch of the cyclotron. State its working principle. Show that the cyclotron frequency is independent of the velocity of the charged particles. (*Delhi 2011C*)
- 27. Explain the principle and working of a cyclotron with the help of a schematic diagram. Write the expression for cyclotron frequency.

LA (5 marks)

- **28.** (a) Deduce an expression for the frequency of revolution of a charged particle in a magnetic field and show that it is independent of velocity or energy of the particle.
 - (b) Draw a schematic sketch of a cyclotron. Explain, giving the essential details of its construction, how it is used to accelerate the charged particles. (AI 2014)
- 29. (a) Draw a schematic sketch of a cyclotron. Explain clearly the role of crossed electric and magnetic field in accelerating the charge. Hence derive the expression for the kinetic energy acquired by the particles.

- (b) An α -particle and a proton are released from the centre of the cyclotron and made to accelerate.
- (i) Can both be accelerated at the same cyclotron frequency? Give reason to justify your answer.
- (ii) When they are accelerated in turn, which of the two will have higher velocity at the exit slit of the dees? (AI 2013)
- 30. Write the expression for the \vec{F} , force, acting on a charged particle of charge 'q', moving with a velocity ν in the presence of both electric field \vec{E} and magnetic field \vec{B} . Obtain the condition under which the particle moves undeflected through the fields. (2/5, AI 2012)
- 31. With the help of a labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it works to accelerate the charged particles. Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason. (Delhi 2011)
- **32.** Draw a schematic sketch of a cyclotron, explain its working principle and deduce the expression for the kinetic energy of the ions accelerated.

(3/5, Foreign 2011)

- **33.** Draw a schematic sketch of a cyclotron. State its working principle. Describe briefly how it is used to accelerate charged particles. Show that the period of a revolution of an ion is independent of its speed or radius of the orbit. Write two important uses of a cyclotron. (AI 2011C)
- **34.** Draw a schematic sketch of a cyclotron. Explain, giving the essential details of its construction, how it is used to accelerate the charged particles. (3/5, AI 2010C)
- 35. Draw a schematic diagram of a cyclotron. Explain its underlying principle and working, stating clearly the function of the electric and magnetic fields applied on a charged particle. Deduce an expression for the period of revolution and show that it does not depend on the speed of the charged particle. (Delhi 2008)
- **36.** Explain with the help of a labelled diagram, the principle and construction of a cyclotron. Deduce an expression for the cyclotron frequency and show that it does not depend on the speed of the charged particle. (AI 2007)

4.5 Magnetic Field Due to a Current Element, Biot-Savart Law

SAI (2 marks)

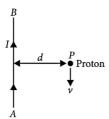
37. State Biot – Savart law in vector form expressing the magnetic field due to an element $\vec{d}l$ carrying current I at a distance \vec{r} from the element.

(1/2, AI 2014C)

38. State Biot-Savart law, giving the mathematical expression for it.

Use this law to derive the expression for the magnetic field due to a circular coil carrying current at a point along its axis. (Delhi 2011)

39. A long straight wire AB carries a current I. A proton P travels with a speed ν , parallel to the wire, at a distance d from it in a direction opposite to the current as shown



in the figure. What is the force experienced by the proton and what is its direction?

LA (5 marks)

40. State Biot-Savart's law and give the mathematical expression for it. (2/5, Delhi 2011)

4.6 Magnetic field on the Axis of a Circular Current Loop

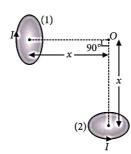
VSA (1 mark)

41. Depict the direction of the magnetic field lines due to a circular current carrying loop.

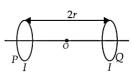
(Delhi 2012C)

SA I (2 marks)

42. Two very small identical circular loops, (1) and (2), carrying equal currents *I* are placed vertically (with respect to the plane of the paper) with their geometrical axes perpendicular to



- each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point *O.* (*Foreign 2014*)
- 43. Two identical circular loops, *P* and *Q*, each of radius *r* and carrying equal currents are kept in the parallel planes



having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O which is equidistant from the loops P and Q. Find the magnitude of the net magnetic field at O.

(Delhi 2012)

44. Two identical coils, each of radius 'R' and number of turns 'N' are lying in perpendicular planes such that their centres coincide. Find the magnitude and direction of the resultant magnetic field at the centre of the coils, if they are carrying currents I and $\sqrt{3}I$ respectively.

(Delhi 2012C)

45. A straight wire of length *L* is bent into a semi-circular loop. Use Biot-Savart law to deduce an expression for the magnetic field at its centre due to the current *I* passing through it.

(Delhi 2011C)

SA II (3 marks)

46. Use Biot-Savart law to derive the expression for the magnetic field on the axis of a current carrying circular loop of radius *R*.

Draw the magnetic field lines due to circular wire carrying current *I*. (AI 2016)

47. Two identical coils *P* and *Q* each of radius *R* are lying in perpendicular planes such that they have a common centre. Find the magnitude and



- direction of the magnetic field at the common centre of the two coils, if they carry currents equal to I and $\sqrt{3}I$ respectively. (Foreign 2016)
- **48.** A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid air by a uniform magnetic field *B*. What is the magnitude of the magnetic field?

(2/3, Foreign 2015)

LA (5 marks)

- **49.** Write, using Biot-Savart law, the expression for the magnetic field \vec{B} due to an element $d\vec{l}$ carrying current I at a distance \vec{r} from it in a vector form.
 - Hence derive the expression for the magnetic field due to a current carrying loop of radius R at a point P distant x from its centre along the axis of the loop. (AI 2015)
- 50. Write any two important points of similarities and differences each between Coulomb's law for the electrostatic field and Biot-Savart's law of the magnetic field. Use Biot-Savart's law to find the expression for the magnetic field due to a circular loop of radius 'r' carrying current 'I', at its centre. (Foreign 2015)
- **51.** Using Biot-Savart's law, derive the expression for the magnetic field in the vector form at a point on the axis of a circular current loop.

(3/5, AI 2013)

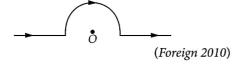
52. State Biot-Savart law, expressing it in the vector form. Use it to obtain the expression for the magnetic field at an axial point, distance 'd' from the centre of a circular coil of radius 'a' carrying current 'l'. Also find the ratio of the magnitudes of the magnetic field of this coil at the centre and at an axial point for which $d = a\sqrt{3}$.

(Delhi 2013C)

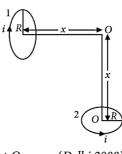
53. State Biot-Savart law. Deduce the expression for the magnetic field due to a circular current carrying loop at a point lying on its axis.

(3/5, AI 2012C)

- **54.** (a) Using Biot-Savart law, deduce an expression for the magnetic field on the axis of a circular current loop.
 - (b) Draw the magnetic field lines due to a current carrying loop.
 - (c) A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown. What is the magnetic field \vec{B} at O due to (i) straight segments (ii) the semi-circular arc?



- 55. (a) Using Biot-Savart's law, derive an expression for the magnetic field at the centre of a circular coil of radius *R*, number of turns *N*, carrying current *i*. (2/5, Delhi 2008)
 - (b) Two small identical circular coils marked 1, 2 carry equal currents and are placed with their geometric axes perpendicular to each other as shown in the figure. Derive an expression for the resultant magnetic field at O.



(Delhi 2008)

56. State Biot-Savart law. Use it to derive an expression for the magnetic field at the centre of a circular loop of radius *R* carrying a steady current *I*. Sketch the magnetic field lines for such a current carrying loop. (3/5, Delhi 2007)

4.7 Ampere's Circuital Law

SAII (3 marks)

- 57. State Ampere's circuital law, expressing it in the integral form. (1/3, *Delhi 2014*)
- **58.** A long straight wire of a circular cross section of radius 'a' carries a steady current 'I'. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point 'r' in the region for (i) r < a and (ii) r > a. (Delhi 2010)

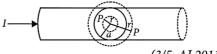
LA (5 marks)

59. Explain how Biot-Savart law enables one to express the Ampere's circuital law in the integral form, viz.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

where I is the total current passing through the surface. (2/5, AI 2015)

60. Figure shows a long straight wire of a circular cross-section of radius 'a' carrying steady current *I*. The current *I* is uniformly distributed across this cross-section. Derive the expressions for the magnetic field in the region r < a and r > a.



(3/5, AI 2011C)

4.8 The Solenoid and the Toroid

VSA (1 mark)

61. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why? (*Delhi 2009*)

SAI (2 marks)

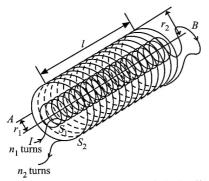
- **62.** Draw the magnetic field lines due to a current passing through a long solenoid. Use Ampere's circuital law, to obtain the expression for the magnetic field due to the current *I* in a long solenoid having *n* number of turns per unit length. (*Delhi 2014C*)
- **63.** A long solenoid of length '*l*' having *N* turns carries a current *I*. Deduce the expression for the magnetic field in the interior of the solenoid.

(AI 2011C)

64. Using Ampere's circuital law, obtain an expression for the magnetic field along the axis of a current carrying solenoid of length *l* and having *N* number of turns. (AI 2008)

SA II (3 marks)

65. Two long coaxial insulated solenoids, S_1 and S_2 of equal lengths are wound one over the other as shown in the figure. A steady current "I" flow through the inner solenoid S_1 to the other end B, which is connected to the outer solenoid S_2 through which the same current "I" flows in the opposite direction so as to come out at end A. If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point (i) inside on the axis and (ii) outside the combined system.



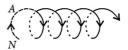
(2/3, Delhi 2014)

- **66.** (a) How is a toroid different from a solenoid?
 - (b) Use Ampere's circuital law to obtain the magnetic field inside a toroid.
 - (c) Show that in an ideal toroid, the magnetic field (i) inside the toroid and (ii) outside the toroid at any point in the open space is zero.

 (AI 2014C)

LA (5 marks)

- 67. (a) State ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius 'r', having 'n' turns per unit length and carrying steady current I.
 - (b) An observer to the left of a solenoid of N turns each of cross section area 'A' observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment m = NIA.



(Delhi 2015)

- **68.** (a) Derive an expression for magnetic field inside, along the axis of an air cored solenoid.
 - (b) Sketch the magnetic field lines for a finite solenoid. How are these field lines different from the electric field lines for an electric dipole? (4/5, Foreign 2010)
- **69.** What does a toroid consist of ? Show that for an ideal toroid of closely wound turns, the magnetic field (i) inside the toroid is constant, and (ii) in the open space inside and exterior to the toroid is zero.

 (3/5, AI 2010C)

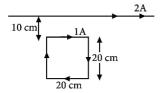
4.9 Force between two Parallel Currents, the Ampere

VSA (1 mark)

70. Using the concept of force between two infinitely long parallel current carrying conductors, define one ampere of current. (AI 2014)

SAI (2 marks)

71. A square loop of side 20 cm carrying current of 1 A is kept near an infinite long straight wire carrying a current of 2 A in the same plane as shown in the figure.



Calculate the magnitude and direction of the net force exerted on the loop due to the current carrying conductor. (AI 2015C)

SA II (3 marks)

- **72.** Two long straight parallel conductors carry steady current I_1 and I_2 separated by a distance d. If the currents are flowing in the same direction, show how the magnetic field set up in one produces an attractive force on the other. Obtain the expression for this force. Hence define one ampere. (*Delhi 2016*)
- 73. (a) Two long straight parallel conductors 'a' and 'b' carrying steady currents Ia and Ib are separated by a distance d. Write the magnitude and direction of the magnetic field produced by the conductor 'a' at the points along the conductor 'b'. If the currents are flowing in the same direction, what is the nature and magnitude of the force between the two conductors?
 - (b) Show with the help of a diagram how the force between the two conductors would change when the currents in them flow in the opposite directions. (Foreign 2014)
- 74. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5 A is held directly above AB at a height of 1 mm. Find the mass per unit length of the wire CD so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in AB. [Take the value of $g = 10 \text{ m s}^{-2}$] (AI 2013)
- 75. Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence define one ampere. (*Delhi 2009*)

LA (5 marks)

76. Two long straight parallel conductors carrying steady currents I_1 and I_2 are separated by a distance 'd'. Explain briefly with the help of a suitable diagram, how the magnetic field due to

- one conductor acts on the other. Hence deduce the expression for the force acting between the two conductors. Mention the nature of this force. (3/5, AI 2012)
- 77. Two long and parallel straight wires carrying currents of 2 A and 5 A in the opposite directions are separated by a distance of 1 cm. Find the nature and magnitude of the magnetic force between them. (2/5, Foreign 2011)

4.10 Torque on Current Loop, Magnetic Dipole

SAI (2 marks)

- 78. A square shaped plane coil of area 100 cm² of 200 turns carries a steady current of 5 A. If it is placed in a uniform magnetic field of 0.2 T acting perpendicular to the plane of the coil. Calculate the torque on the coil when its plane makes an angle of 60° with the direction of the field. In which orientation will the coil be in stable equilibrium? (AI 2015C)
- **79.** Circular coil of *N* turns and radius *R* carries a current *I*. It is unwound and rewound to make another coil of radius *R*/2. Current *I* remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil.

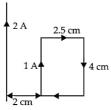
 (AI 2012)
- **80.** A short bar magnet of magnetic moment 0.9 J/T is placed with its axis at 30° to a uniform magnetic field. It experiences a torque of 0.063 I.
 - (i) Calculate the magnitude of the magnetic field.
 - (ii) In which orientation will the bar magnet be in stable equilibrium in the magnetic field? (Foreign 2012)
- **81.** Deduce the expression for the magnetic dipole moment of an electron orbiting around the central nucleus. (AI 2010)
- **82.** A square coil of side 10 cm has 20 turns and carries a current of 12 A. The coil is suspended vertically; the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field. If the torque experienced by the coil equal 0.96 N-m, find the magnitude of the magnetic field. (*Delhi 2010C*)

SA II (3 marks)

83. A closely wound solenoid of 2000 turns and cross sectional area 1.6×10^{-4} m² carrying a current of 4.0 A is suspended through its centre allowing it to turn in a horizontal plane. Find (i) the magnetic moment associated with the solenoid, (ii) the torque on the solenoid if a horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid.

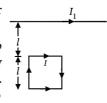
(AI 2015C)

84. A rectangular loop of wire of size 2.5 cm × 4 cm carries a steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and



the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of the force on the loop due to the current carrying wire. (*Delhi 2012*)

85. Write the expression for the magnetic moment (m) due to a planar square loop of side 'l' carrying a steady current I in a vector form. In the given figure this loop



is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance l as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop.

LA (5 marks)

86. Derive the expression for the torque on a rectangular current carrying loop suspended in a uniform magnetic field.

(2/5, Delhi 2013, AI 2009)

- 87. A rectangular loop of size $l \times b$ carrying a steady current I is placed in a uniform magnetic field \vec{B} . Prove that the torque acting on the loop is given by $\vec{\tau} = \vec{m} \times \vec{B}$, where \vec{m} is the magnetic moment of the loop. (2/5, AI 2012)
- **88.** (a) Show that a planar loop carrying a current *I*, having *N* closely wound turns and area of

- cross-section A, possesses a magnetic moment $\vec{m} = NI \vec{A}$.
- (b) When this loop is placed in a magnetic field \vec{B} , find out the expression for the torque acting on it. (3/5, Foreign 2011)

4.11 The Moving Coil Galvanometer

VSA (1 mark)

- **89.** Write the underlying principle of a moving coil galvanometer. (*Delhi 2016*)
- 90. Why should the spring/ suspension wire in a moving coil galvanometer have low torsional constant? (AI 2008)

SAI (2 marks)

91. A galvanometer has a resistance of 30 Ω . It gives full scale deflection with a current of 2 mA. Calculate the value of the resistance needed to convert it into an ammeter of range 0 – 0.3 A.

(AI 2007)

SA II (3 marks)

- **92.** State the principle of working of a galvanometer.
 - A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto V/2 volts. Find the resistance, in terms of R_1 and R_2 , required to be connected to convert it into a voltmeter that can read upto 2 V. Also find the resistance G of the galvanometer in terms of R_1 and R_2 . (Delhi 2015)
- 93. (a) Why is the magnetic field radial in a moving coil galvanometer? Explain how it is achieved.
 - (b) A galvanometer of resistance 'G' can be converted into a voltmeter of range (0-V) volts by connecting a resistance 'R' in series with it. How much resistance will be required to change its range from 0 to V/2? (AI 2015C)
- **94.** (a) Define the current sensitivity of a galvanometer.
 - (b) The coil area of a galvanometer is 16×10^{-4} m². It consists of 200 turns of a wire and is in a magnetic field of 0.2 T. The restoring torque

- constant of the suspension fibre is 10^{-6} N m per degree. Assuming the magnetic field to be radial, calculate the maximum current that can be measured by the galvanometer if the scale can accommodate 30° deflection. (AI 2013C)
- **95.** Draw a labelled diagram of a moving coil galvanometer and explain its working. What is the function of radial magnetic field inside the coil? (*Foreign 2012*)
- 96. State the underlying principle of working of a moving coil galvanometer. Write two reasons why a galvanometer can not be used as such to measure current in a given circuit. Name any two factors on which the current sensitivity of a galvanometer depends. (*Delhi 2010*)
- 97. A moving coil galvanometer of resistance G, gives its full scale deflection when a current I_G flows through its coil. It can be converted into an ammeter of range (0 to I) ($I > I_G$) when a shunt of resistance S is connected across its coil. If this galvanometer is converted into an ammeter of range 0 to 2I, find the expression for the shunt required in terms of S and G. (*Delhi 2010C*)

LA (5 marks)

- **98.** (i) With the help of a neat and labelled diagram. explain the principle and working of a moving coil galvanometer.
 - (ii) What is the function of uniform radial field and how is it produced?
 - (iii) Define current sensitivity of a galvanometer. How is current sensitivity increased?

(Foreign 2016)

- **99.** (a) Explain using a labelled diagram, the principle and working of a moving coil galvanometer. What is the function of (i) uniform radial magnetic field, (ii) soft iron core?
 - (b) Define the terms (i) current sensitivity and (ii) voltage sensitivity of a galvanometer. Why does increasing the current sensitivity not necessarily increase voltage sensitivity?

(AI 2015)

- **100.** (a) Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.
 - (b) Answer the following:
 - (i) Why is it necessary to introduce a cylindrical

soft iron core inside the coil of a galvanometer? (ii) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain, giving reason.

(AI 2014)

- 101. (a) State using a suitable diagram, the working principle of a moving coil galvanometer. What is the function of a radial magnetic field and the soft iron core used in it?
 - (b) For converting a galvanometer into an ammeter, a shunt resistance of small value is used in parallel, whereas in the case of a voltmeter a resistance of large value is used in series. Explain why.

 (4/5, Delhi 2013C)
- **102.** (a) Explain briefly with the help of a labelled diagram, the principle and working of a moving coil galvanometer.
 - (b) Define the term 'current sensitivity' of a galvanometer. How is it that increasing current sensitivity may not necessarily increase its voltage sensitivity? Explain. (AI 2012C)
- 103.A galvanometer coil of 50 Ω resistance shows full scale deflection for a current of 5 mA. How will you convert this galvanometer into a voltmeter of range 0 to 15 V? (2/5, Foreign 2011)
- **104.** (a) With the help of a diagram, explain the principle and working of a moving coil galvanometer.
 - (b) What is the importance of a radial magnetic field and how is it produced?
 - (c) Why is it that while using a moving coil galvanometer as a voltmeter a high resistance in series is required whereas in an ammeter a shunt is used?

 (AI 2010)
- 105. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By how much faster does its voltage sensitivity change? (2/5, AI 2008)
- 106. (a) Draw a labelled diagram of a moving coil galvanometer. State the principle on which it works.
 - (b) Deduce an expression for the torque acting on a rectangular current carrying loop kept in a uniform magnetic field. Write two factors on which the current sensitivity of a moving coil galvanometer depend. (Delhi 2007)

Detailed Solutions

1. The magnetic force experienced by the charge q moving with velocity \vec{v} in magnetic field \vec{B} is given by Lorentz force, $\vec{F} = q(\vec{v} \times \vec{B})$

The direction of the Lorentz force is perpendicular to the plane containing \vec{v} and \vec{B} . Its direction is given by right-handed screw rule.

2. One tesla is defined as the magnitude of magnetic field which produces a force of 1 newton when a charge of 1 coulomb moves perpendicularly in the region of the magnetic field at a velocity of 1 m/s.

$$F = qvB \Rightarrow B = \frac{F}{qv}$$
 or $1 \text{ T} = \frac{1 \text{ N}}{(1 \text{ C})(1 \text{ m/s})}$

3. Given, $\vec{F} = q(\vec{v} \times \vec{B})$

 $\Rightarrow F = qvB \sin \theta$

where, θ is the angle between \vec{v} and \vec{B} .

$$\Rightarrow B = \frac{F}{qv\sin\theta}$$

i.e. if q = 1 C, v = 1 m/s, $\theta = 90^{\circ}$, then B = F The magnetic field at any point is given by

$$B = \frac{F}{qv\sin\theta} = \frac{1N}{((1C)(1m/s)\sin 90^\circ)}$$

= 1N/A-m = 1T

:. SI unit of magnetic field = tesla (T)

Thus, the magnetic field induction at a point is said to be one tesla if a charge of one coulomb while moving at right angle to a magnetic field with a velocity of 1 m/s experiences a force of 1 N at that point.

- **4.** $\vec{F} = q(\vec{v} \times \vec{B})$, i.e., $F = qvB \sin\theta$
- (i) Force will be maximum for $\sin \theta = 1$, *i.e.*, charge is moving normal to the magnetic field.
- (ii) Force will be minimum for $\sin\theta = 0$, *i.e.*, charge is moving either parallel or antiparallel to the magnetic field.
- 5. (a) Values displayed by Kamal:
- (i) Being educated, he knows about MRI (magnetic resonance imaging).
- (ii) Took prompt decisions to take the help of his family, friends and neighbours and arranged the cost of MRI.
- (iii) He showed his empathy, helping attitude and caring nature for his uncle.
- (b) Magnetic force on moving charge particle in uniform magnetic field \vec{B} can be given as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

or $|F| = qvB \sin \theta$

(i) Maximum force at $\theta = 90^{\circ}$

$$F = avB$$

 $= 1.6 \times 10^{-19} \times 10^4 \times 0.1$

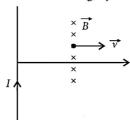
$$= 1.6 \times 10^{-16} \,\mathrm{N}$$

(ii) Minimum force at $\theta = 0^{\circ}$ and 180°

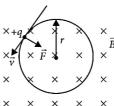
F = 0

i.e., force is minimum when the charge particle either move parallel or antiparallel to the magnetic field lines.

- 6. \vec{F}_B is always perpendicular to \vec{v} and \vec{B} , and cannot change the speed of particle. In other words, magnetic force cannot speed up or slow down a charged particle. Hence, kinetic energy remains unaffected.
- 7. From relation $\vec{F} = qvB[\hat{i} \times (-\hat{k})] = +qvB(\hat{j})$ Magnetic force \vec{F} will be along + y axis.



8. The charged particle will move along a circular path.



9. Charge on deuteron (q_d) = charge on proton (q_p)

Radius of circular path
$$(r) = \frac{p}{Bq} \left(\because qvB = \frac{mv^2}{r} \right)$$

 $r \propto \frac{1}{q}$ [for constant momentum (p)]

So,
$$\frac{r_p}{r_d} = \frac{q_d}{q_p} = 1$$

Hence, $r_p : r_d = 1 : 1$

10.
$$\frac{F_A}{F_B} = \frac{q_1 |(\vec{v}_1 \times \vec{B}_1)|}{q_2 |(\vec{v}_2 \times \vec{B}_2)|}$$

$$\frac{1}{2} = \frac{q v_1 B \sin 90^{\circ}}{(2q) v_2 B \sin 90^{\circ}}, [\because B_1 = B_2; \text{ magnetic field is same}).$$

$$1 = \frac{v_1}{v_2}$$

$$\therefore v_1 : v_2 = 1 : 1$$

11. From Lorentz force, $F = q(\vec{v} \times \vec{B})$

$$\Rightarrow F \hat{j} = q(v\hat{i} \times \vec{B})$$

It is clear that \vec{B} is along -z axis, as $\hat{i} \times (-\hat{k}) = \hat{j}$

12. :
$$\vec{F} = q (\vec{v} \times \vec{B})$$

Magnetic force is always perpendicular to both \vec{v} and \vec{B} .

13. From relation $\vec{F} = q(\vec{v} \times \vec{B})$

$$\vec{F} = -qvB(\hat{i} \times \hat{j}) = -qvB(\hat{k})$$

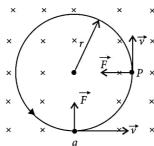
Magnetic force \vec{F} will be along -z axis.

14. When a particle of charge 'q' and mass 'm' is directed to move perpendicular to the uniform magnetic field ' \vec{B} ' with velocity ' \vec{v} ',

the force on the charge

$$\vec{F} = q(\vec{v} \times \vec{B})$$

This magnetic force always acts perpendicular to the velocity of charged particle. Hence magnitude of velocity remains constant but direction changes continuously. Consequently the path of the charged particle in a perpendicular magnetic field becomes circular. The magnetic force (*qvB*) provides the necessary centripetal force to move along a circular path.



Then,
$$qvB = \frac{mv^2}{r} \implies r = \frac{mv}{qB}$$

Here r = radius of the circular path followed by the charge.

15. Charge on deuteron (q_d) = charge on proton (q_p) Radius of circular path $r \propto m \left(\because qvB = \frac{mv^2}{r}\right)$

(For constant velocity ν)

$$\frac{r_d}{r_p} = \frac{m_d}{m_p}$$

$$\therefore m_d = 2m_p$$

$$\Rightarrow r_d = 2r_p \text{ or } r_d : r_p = 2 : 1$$

$$\times \times \times \times \times \times$$
Proton
$$\times \times \times \times \times \times$$
Deuteron
$$\times \times \times \times \times \times$$

16. Magnetic Lorentz force, $\vec{F}_m = q(\vec{v} \times \vec{B})$

... $W = Fd \cos 90^\circ = 0$ [: F and displacement are perpendicular to each other]

No work is done by magnetic Lorentz force on the charged particle.

17. Radius of charged particle in magnetic field

$$r = \frac{mv}{qB}$$
, $r \propto \frac{m}{q}$ for same v and B .

$$\frac{r_p}{r_\alpha} = \frac{(m/q)_p}{(m/q)_\alpha} = \frac{(m_p/e)}{(4m_p/2e)} = \frac{1}{2}$$

$$\frac{\alpha}{x} \times x \times x \times x$$

$$\frac{\alpha}{p} \times x \times x \times x \times x$$

18. (a) Magnetic force acting on a charged particle q moving with a velocity v in a uniform magnetic field B is given by

$$\vec{F} = q (\vec{v} \times \vec{B})$$

(b) Magnetic force on α-particle

$$\vec{F}_{\alpha} = q\vec{v} \times \vec{B} = 2evB$$
 upward

So, curve will bend upwards as force is perpendicular to the velocity.

Magnetic force on neutron, F = 0 (as q = 0)

So, neutron will move along straight line.

Magnetic force on electron

$$\vec{F}_{e} = q \vec{v} \times \vec{B} = |-evB|$$
 downwards

So, curve will bend downwards as force is perpendicular to the velocity,

For a charged particle moving in a uniform magnetic field \vec{B} perpendicular to velocity,

$$qvB = \frac{mv^2}{r} \Longrightarrow r = \frac{mv}{qB}$$

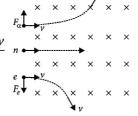
r is the radius of curved path.

Here
$$v_{\alpha} = v_n = v_e = v$$

Radius of path traced by

$$\alpha$$
-particle, $r_{\alpha} = \frac{4m_e v}{2e B} = \frac{2m_e v}{e B}$

Radius of path traced by electron, $r_e = \frac{m_e v}{\rho R}$



19. Magnetic force on a charged particle

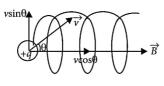
$$\vec{F} = q(\vec{v} \times \vec{B}) : |\vec{F}| = qvB\sin\theta$$

Thus radius of circular path

$$r = \frac{mv\sin\theta}{qB}$$

Time period,

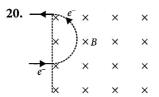
$$T = \frac{2\pi m}{Bq}$$



Horizontal distance moved by the particle in one rotation,

pitch =
$$v \cos \theta \times T = \frac{2\pi m}{Bq} v \cos \theta$$

Path of the charged particle will be helical.



Trajectory of electron

Let the time taken by the electron to come out of the region of magnetic field be t.

Velocity of the electron, $v = 4 \times 10^4$ m/s

Magnetic field, $B = 10^{-5} \text{ T}$

Mass of the electron, $m = 9 \times 10^{-31}$ kg

We know

$$t = \frac{\pi r}{v}$$
where $r = \frac{mv}{qB}$

$$t = \frac{\pi m}{Bq} = \frac{3.14 \times 9 \times 10^{-31}}{10^{-5} \times 1.6 \times 10^{-19}}$$

$$\Rightarrow t = 17.66 \times 10^{-7} \text{ s} = 1.77 \text{ µs}$$

Thus, the time taken by the electron to come out of the region of magnetic field is 1.77 µs.

21. We know, Lorentz force, $F = Bqv \sin \theta$ where θ = angle between velocity of particle and magnetic field = 90°

So, Lorentz force, F = Bqv

Thus the particles will move in circular path.

$$Bqv = \frac{mv^2}{r} \implies r = \frac{mv}{Ba}$$

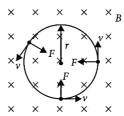
 $Bqv = \frac{mv^2}{r} \implies r = \frac{mv}{Bq}$ Let $m_p = \text{mass of proton}, m_d = \text{mass of deuteron},$ v_p = velocity of proton and v_d = velocity of deuteron The charge of proton and deuteron are equal. Given that $m_p v_p = m_d v_d$

$$r_p = \frac{m_p v_p}{Ba} \qquad \dots (i)$$

$$r_d = \frac{m_d v_d}{Ba} \qquad ...(ii)$$

As (i) and (ii) are equal, so $r_p = r_d = r$

Thus, the trajectory of both the particles will be same.



22. The charged particle moves in a circular path with a constant speed and is acted upon only by the magnetic field. The radius of the circular path is given by

$$qvB = mv^2/r$$

or r = mv/qB

Period of revolution,

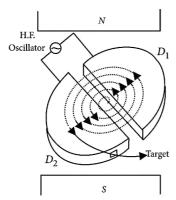
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Frequency of revolution, $f = \frac{1}{T} = \frac{qB}{2\pi m}$

Clearly, frequency f is independent of both v and rand is also independent of energy.

- 23. The charged particle goes undeflected in the presence of crossed electric and magnetic fields only when both these fields are perpendicular to velocity of charged particle. In that case, qE = qvB.
- 24. Cyclotron: It is a device by which positively charged particles like protons, deuterons, etc. can be accelerated.

Principle: A positively charged particle can be accelerated by making it to cross the same electric field repeatedly with the help of a magnetic field.



Working and theory: At a certain instant, let D_1 be positive and D_2 be negative. The radius of the circular path is given by

 $qvB = mv^2/r$

or r = mv/qB

Period of revolution,

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

 $T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$ Frequency of revolution, $f = \frac{1}{T} = \frac{qB}{2\pi m}$

Clearly, frequency f is independent of both ν and rand is called cyclotron frequency. If the frequency of applied a.c. is equal to f, then every time the proton reaches the gap between the dees, the direction of electric field is reversed and proton receives a push and finally it gains very high kinetic energy. The accelerated protons are deflected towards the target.

25. (a) When a charged particle having charge q moves inside a magnetic field \vec{B} with velocity \vec{v} , it experiences a force

$$\vec{F} = q(\vec{v} \times \vec{B})$$

When \vec{v} is perpendicular to \vec{B} , the force \vec{F} on the charged particle provides the centripetal force and makes it move along a circular path.

The point charge travels in the plane perpendicular to both \vec{v} and \vec{B} .

(b) If a component of velocity of the charge particle is parallel to the direction of the magnetic field, then the force experienced due to that component will be zero, because

 $F = qvB \sin 0^{\circ} = 0$ and particle will move in straight line. Also, the force experienced by the component perpendicular to \vec{B} moves the particle in a circular path. The combined effect of both the components will move the particle in a helical path.

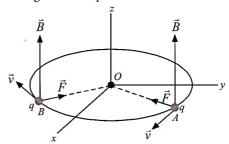
(c) The direction of the magnetic force is along negative Y-axis and so, the direction of electric force should be along the positive Y-axis to counter balance the magnetic force and than the charge particle will move in the straight line path.

Therefore, the direction of electric field is along the positive *Y*-axis and its magnitude is given by E = vB.

- 26. Refer to answer 24.
- 27. Refer to answer 24.
- 28. (a) When a charged particle with charge q moves inside a magnetic field with velocity v, it experiences a force, which is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

If \vec{v} is perpendicular to \vec{B} , the force on the charged particle provides the centripetal force and makes it move along a circular path.



Let *m* be the mass of the charged particle and *r* be the radius of the circular path

$$\therefore q |(\vec{v} \times \vec{B})| = \frac{mv^2}{r}$$

 ν and B are at right angles

$$\therefore qvB = \frac{mv^2}{r}, r = \frac{mv}{Bq}$$

Time period of circular motion of the charged particle can be calculated as

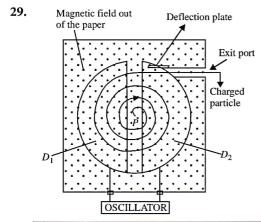
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{Bq} = \frac{2\pi m}{Bq}$$
Angular frequency is $\omega = \frac{2\pi}{T}$

$$\omega = \frac{Bq}{m}$$

Therefore, the frequency of the revolution of the charged particle is independent of the velocity or the energy of the particle.

(b) Refer to answer 24.

Construction: It consists of two semi-cylindrical boxes D_1 and D_2 , called dees enclosed in an evacuated chamber. The chamber is kept between the poles of a powerful magnet so that uniform magnetic field acts perpendicular to the plane of the dees. An alternating voltage is applied in the gap between the two dees by using a high frequency oscillator. The electric field is zero inside the dees.



(a) Electric field accelerates the particle when it passes through the gap and imparts energy to charged particle. Magnetic field makes the charged particle to move in semi circular paths.

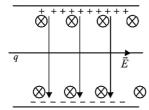
Velocity of particle

$$v = \frac{Bqr}{m}$$

$$\therefore K = \frac{1}{2}mv^2 = \frac{B^2q^2r^2}{2m}$$

- (b) (i) No. The cyclotron frequency depends on the mass of the particle.
- (ii) Proton.
- **30.** In presence of electric field and magnetic field, the net force on a moving charged particle is called Lorentz force given by $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$.

If a charged particle passes through a region of uniform mutually perpendicular electric and magnetic fields undeflected, then



force due to magnetic field = force due to electric field

or
$$qvB = qE$$
 or $v = \frac{E}{R}$

that gives the velocity of charged particle.

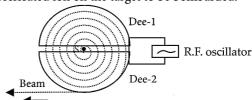
31. Refer to answer 24.

When charged particle reaches near the periphery of dee, it is moving in a circular path of maximum radius equal to radius R of dee and posses maximum kinetic energy

$$KE_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}m\frac{q^2B^2R^2}{m^2} = \frac{q^2B^2R^2}{2m}$$

when it is extracted from dees at point N.

32. Principle: The positive ions produced from a source are accelerated. Due to the presence of perpendicular magnetic field, the ion will move in a circular path. The phenomenon is continued till the ion reaches at the periphery where an auxiliary negative electrode (deflecting plate) deflects the accelerated ion on the target to be bombarded.



Expression for kinetic energy attained:

If *R* be the radius of the path and v_{max} the velocity of the ion when it leaves the periphery, then

$$v_{\text{max}} = \frac{qBR}{m}$$

The kinetic energy of the ion when it leaves the apparatus is.

$$K.E. = \frac{1}{2}mv_{\text{max}}^2 = \frac{q^2B^2R^2}{2m}$$

When charged particle crosses the gap between dees it gains KE = qV

In one revolution, it crosses the gap twice, therefore if it completes n-revolutions before emerging the dees, the kinetic energy gained = 2nqV

Thus,
$$K.E. = \frac{q^2 B^2 R^2}{2m} = 2nqV$$

33. Refer to answer 24.

Uses:

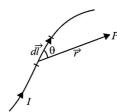
- (i) The cyclotron is used in hospitals to produce radioactive substances which can be used in diagnosis and for treatment purposes.
- (ii) It is used to implant ions into solids and modify their properties or even synthesise new materials.
- 34. Refer to answer 28(b).

35. Refer to answer 28.

36. Refer to answer 28.

37. A current carrying wire produces a magnetic field around it. Biot-Savart law states that magnitude of intensity of small magnetic field $d\vec{B}$ due to current I carrying element $d\vec{l}$ at any point P at distance r from it is given by

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$



where θ is the angle between \vec{r} and $d\vec{l}$ and $\mu_0 = 4\pi \times 10^{-7}~T$ m A^{-1} is called permittivity of free space.

In vectorial form,

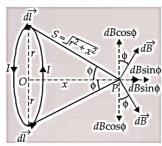
$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi} \frac{d\overrightarrow{l} \times \overrightarrow{r}}{r^3}$$

So, the direction of $d\vec{B}$ is perpendicular to the plane containing \vec{r} and $d\vec{l}$.

S.I. unit of magnetic field strength is tesla denoted by 'T' and cgs unit is gauss denoted by 'G', where $1 T = 10^4 G$

38. Refer to answer 37.

Magnetic field on the axis of circular coil



Small magnetic field due to current element of circular coil of radius r at point P at distance x from its centre is

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^{\circ}}{S^2} = \frac{\mu_0}{4\pi} \frac{Idl}{(r^2 + x^2)}$$

Component $dB\cos\phi$ due to current element at point P is cancelled by equal and opposite component

 $dB\cos\phi$ of another diametrically opposite current element, whereas the sine components $dB\sin\phi$ add up to give net magnetic field along the axis. So, net magnetic field at point P due to entire loop is

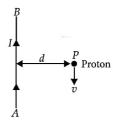
$$B = \oint dB \sin \phi = \int_{0}^{2\pi r} \frac{\mu_0}{4\pi} \frac{Idl}{(r^2 + x^2)} \cdot \frac{r}{(r^2 + x^2)^{1/2}}$$

$$B = \frac{\mu_0 Ir}{4\pi (r^2 + x^2)^{3/2}} \int_0^{2\pi r} dl \text{ or } B = \frac{\mu_0 Ir}{4\pi (r^2 + x^2)^{3/2}} 2\pi r$$

or
$$B = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$
 directed along the axis,

- (a) towards the coil if current in it is in clockwise direction
- (b) away from the coil if current in it is in anticlockwise direction.

39.



Magnetic field due to I at P

 $B = \frac{\mu_0 I}{2\pi d}$ into the plane of the paper.

Expression for Lorentz magnetic force

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$= e\left(\vec{v} \times \frac{\mu_0 I}{2\pi d} \hat{n}\right)$$

$$\vec{F} = \frac{\mu_0 IeV}{2\pi d} \text{ away from the wire}$$

- **40.** Refer to answer 37.
- **41.** Magnetic field lines due to circular wire carrying current *I*:



42. The magnetic field at an axial point due to a circular loop is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + r^2)^{3/2}}$$

where

I = current through the loop

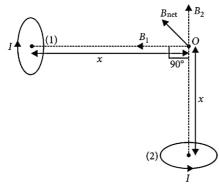
a = radius of the loop

r = distance of O from the centre of the loop.

Since I, a and r = x are the same for both the loops, the magnitude of B will be the same and is given by

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$

The direction of magnetic field due to loop (1) will be away from *O* and that due to loop (2) will be towards *O* as shown. The direction of the net magnetic field will be as shown below:



The magnitude of the net magnetic field is given by

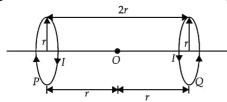
$$B_{\rm net} = \sqrt{B_1^2 + B_2^2}$$

$$B_{\text{net}} = \frac{\mu_0}{4\pi} \frac{2\sqrt{2\pi}Ia^2}{(a^2 + x^2)^{3/2}}$$

43. The magnetic field at a point on the axis of a circular current carrying loop is

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

where *R* is the radius of the loop, *x* is the distance of the point on the axis from the centre of the loop.



Magnetic field at O due to the loop P is

$$B_P = \frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 I r^2}{2(2r^2)^{3/2}} = \frac{\mu_0 I}{2(2)^{3/2} r}$$

By the right hand thumb's rule, the direction of the

magnetic field will be towards left. Magnetic field at *O* due to the loop *Q* is

$$B_Q = \frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 I r^2}{2(2r^2)^{3/2}} = \frac{\mu_0 I}{2(2)^{3/2} r}$$

By the right hand thumb's rule, the direction of the magnetic field will be towards left.

Since B_P and B_Q are equal in magnitude and in same direction. Therefore net magnetic field at O is

$$\vec{B} = \vec{B}_P + \vec{B}_Q = \frac{\mu_0 I}{2^{3/2} r}$$
 towards left.

44. Magnetic field at the centre of the coils due to coil *P*, having current *I* is

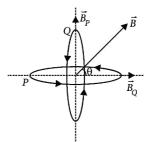
$$B_P = \frac{\mu_0 I}{2R}$$

And magnetic field due to coil Q having current $\sqrt{3}I$ is

$$B_Q = \frac{\mu_0 \sqrt{3}I}{2R}$$

Since both coils are inclined to each other at an angle of 90°, the magnitude of their resultant magnetic field at the common centre will be

$$B = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0 I}{2R} \sqrt{1 + 3} = \frac{\mu_0 I}{R}$$



The directions of B_P and B_Q are as indicated in the figure. The direction of the resultant field is at an angle θ given by

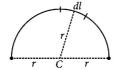
$$\theta = \tan^{-1} \left(\frac{B_P}{B_Q} \right) = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^{\circ}$$

Hence, the direction of the magnetic field will be at an angle 30° to the plane of loop *P*.

45. Let length *L* is bent into semi-circular loop. Length of wire = Circumference of semi circular wire $\Rightarrow L = \pi r$

$$r = \frac{L}{\pi} \qquad ...(i)$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C,



$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^{\circ}}{r^2} \qquad [\because I \overrightarrow{dl} \perp \overrightarrow{r}, \therefore \theta = 90^{\circ}]$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$$

.. Net magnetic field at C due to semi-circular

$$B = \int_{\text{semicircle}} \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_{\text{semicircle}} dl$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} L$$

But,
$$r = \frac{L}{\pi}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2} = \frac{\mu_0}{4\pi} \times \frac{IL}{L^2} \times \pi^2$$

$$\Rightarrow B = \frac{\mu_0 I \pi}{4L}$$

- 46. Refer to answers 38 and 41.
- 47. Refer to answer 44.
- **48.** Mass of wire, m = 200 g = 0.2 kg, length of wire l = 1.5 m, current in the wire I = 2 A

In the equilibrium position, the net force on the wire will be zero.

Thus, mg = BIl

$$\Rightarrow B = \frac{mg}{Il}$$

$$\Rightarrow B = \frac{0.2 \times 9.8}{2 \times 1.5}$$

$$\Rightarrow B = 0.65 \text{ T}$$

- 49. Refer to answer 38.
- 50. Similarities between Coulomb's law and Biot-Savart's law:
- 1. The principle of superposition is applicable to both magnetic field \vec{B} as well as electric field \vec{E} .
- 2. Both depend inversely on the square of the

distance from the source to the point of interest. Differences between Coulomb's law and Biot-Savart's law:

- There is an angle dependence in Biot-Savart's law, which is not present in the electrostatic case.
- The electrostatic field is produced by a scalar source, the charge q. However the magnetic field is produced by a vector source $I\overline{dl}$.

According to Biot-Savart's law, the magnetic field due to a current element dl at the observation point

whose position vector is
$$\vec{r}$$
 is given by $\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}}{r^3}$

where μ_0 is the permeability of free space. Consider a circular loop of wire of radius r carrying a current I. Consider a current element dl of the loop.

The direction of dl is along the tangent, so dl $\perp r$. From Biot Savart's law, magnetic field at the centre O due to this current element is

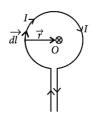
$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin 90^{\circ}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dl}{r^2}$$

The magnetic field due to all suchcurrent elements will point into the plane of paper at the centre O. Hence the total magnetic field at the centre O is

$$B = \int dB = \int \frac{\mu_0 I dl}{4\pi r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2} \int dl = \frac{\mu_0 I}{4\pi r^2} .l$$

$$= \frac{\mu_0 I}{4\pi r^2} .2\pi r \text{ or } B = \frac{\mu_0 I}{2r} .$$



- 51. Refer to answer 38.
- 52. Refer to answer 38.

Magnetic field induction at the centre of the circular coil carrying current is $B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a}$

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi}{a}$$

Magnetic field at an axial point at a distance *d*,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{\left(a^2 + d^2\right)^{3/2}}$$

$$\frac{B_1}{B_2} = \frac{(a^2 + d^2)^{3/2}}{a^3}$$

$$\frac{B_1}{B_2} = \frac{(a^2 + 3a^2)^{3/2}}{a^3} = \frac{(4a^2)^{3/2}}{a^3} \quad [\because d = a\sqrt{3}]$$

$$\frac{B_1}{B_2} = \frac{8}{1}.$$

53. Refer to answer 38.

54. (a) Refer to answer 38.

(b) Refer to answer 41.

(c) Magnetic field due to a current carrying element,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \frac{I\overrightarrow{dl} \times \overrightarrow{r}}{r^3}$$

(i) For straight segments, $\theta = 0$ or π

$$\Rightarrow \vec{dl} \times \vec{r} = dl r \sin 0 \hat{n} = 0$$

$$\therefore B_1 = 0$$

(ii) For semicircular arc, $\Sigma dl = \pi r$, $\theta = \frac{\pi}{2}$

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{\Sigma I \overline{dl} \times \overline{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{I \Sigma dl \sin \frac{\pi}{2}}{r^2} \hat{n} = \frac{\mu_0}{4\pi} \frac{I \pi r}{r^2} \hat{n}$$
$$= \frac{\mu_0 I}{4r} \hat{n}$$

directed perpendicular to plane of paper downward.

55. (a) Refer to answer 50.

For a coil of *N* turns, $B = \frac{\mu_0 Ni}{2R}$

(b) Refer to answer 42.

56. Refer to answers 37, 50 and 41.

57. Ampere's circuital law states that line integral of magnetic field over a closed loop or circuit is μ_0 times the total current *I* threading through the loop *i.e.*,



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

58. (i) Let us consider a circular loop L_1 of radius r_1 ($r_1 < a$) inside the current carrying wire. The current enclosed by this loop L_1 is

$$I_1 = \frac{I}{\pi a^2} \times \pi r_1^2 = \frac{Ir_1^2}{a^2}$$

So by Ampere's circuital law

$$\oint_{L_1} \vec{B} \cdot \vec{dl} = \mu_0 I_1 = \frac{\mu_0 I r_1^2}{a^2} \qquad ...(v)$$

Also,
$$\oint_{L_1} \vec{B} \cdot d\vec{l} = \oint_{L_1} \vec{B} \cdot dl \cos 0^\circ = B \oint_{L_1} dl = B.2\pi r_1$$
 ...(vi)

So by equations (v) and (vi)

$$B.2\pi r_1 = \frac{\mu_0 I r_1^2}{a^2}$$
 or $B_{\text{in}} = \frac{\mu_0 I r_1}{2\pi a^2}$

(ii) Let us consider a solid metallic wire of cross-section radius 'a' carrying current I. Let us consider a circular loop L of radius r outside the wire, representing a magnetic field line. So, at any point on it magnetic field B is along the tangent to field line at that point.

$$\oint_{L} \vec{B} \cdot \vec{dl} = \oint_{L} Bdl \cos 0^{\circ} = B \oint_{L} dl = B.2\pi r \qquad \dots (i)$$

But, by Ampere's circuital law

$$\oint_L \vec{B} \cdot \vec{dl} = \mu_0 I \qquad ...(ii)$$

By equations (i) and (ii), we get

 $B.2\pi r = \mu_0 I$

or
$$B_{\text{out}} = \frac{\mu_0 I}{2\pi r}$$
 ...(iii)

whereas on the surface of current carrying wire

$$B_{\text{surface}} = \frac{\mu_0 I}{2\pi a} \qquad \dots \text{(iv)}$$

59. Ampere's circuital law states that line integral of magnetic field over a closed loop or circuit is μ_0 times the total current I threading through the loop i.e., $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

Proof:

For the small element,

 $\vec{B} \cdot d\vec{l} = Bdl \cos 0^{\circ}$

$$\int \vec{B} \cdot d\vec{l} = \int Bdl = \int \frac{\mu_0}{4\pi} \frac{2I}{r} dl$$

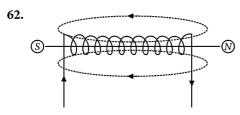
$$= \frac{\mu_0}{4\pi} \frac{2I}{r} \int dl = \frac{\mu_0}{4\pi} \frac{2I}{r} \times 2\pi r$$

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$

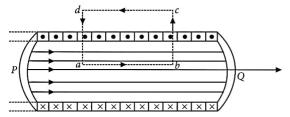
- 60. Refer to answer 58.
- **61.** At the edges of the solenoid, the field lines get diverged due to other fields and non-availability of dipole loops, while in toroid the dipoles orient continuously.







Consider a rectangular amperian loop *abcd* near the middle of solenoid as shown in figure where PQ = l.



Let the magnetic field along the path ab be B and is zero along cd. As the paths bc and da are perpendicular to the axis of solenoid, the magnetic field component along these paths is zero. Therefore, the path bc and da will not contribute to the line integral of magnetic field B.

Total number of turns in length l = nl

The line integral of magnetic field induction *B* over the closed path *abcd* is

$$\int_{abcd} \vec{B} \cdot d\vec{l} = \int_{a}^{b} \vec{B} \cdot d\vec{l} + \int_{b}^{c} \vec{B} \cdot d\vec{l} + \int_{c}^{d} \vec{b} \cdot d\vec{l} + \int_{d}^{a} \vec{B} \cdot d\vec{l}$$

$$\therefore \int_{a}^{b} \vec{B} \cdot d\vec{l} = \int_{a}^{b} B \, dl \cos 0^{\circ} = Bl$$
and
$$\int_{b}^{c} \vec{B} \cdot d\vec{l} = \int_{b}^{c} B \, dl \cos 90^{\circ} = 0 = \int_{d}^{a} \vec{B} \cdot d\vec{l}$$
Also
$$\int_{c}^{d} \vec{B} \cdot d\vec{l} = 0$$

$$\therefore \int_{abcd} \vec{B} \cdot d\vec{l} = Bl + 0 + 0 + 0 = Bl \qquad ...(i)$$

Using Ampere's circuital law

$$\int_{abcd} \vec{B} \cdot d\vec{l} = \mu_0 \times \text{total current in rectangle } abcd$$

$$= \mu_0 \times \text{number of turns in rectangle} \times \text{current}$$

$$= \mu_0 \times nl \times I = \mu_0 \, nl \, I \qquad ...(ii)$$
From (i) and (ii), we have
$$Bl = \mu_0 nl \, I$$

$$\therefore B = \mu_0 nl$$

It gives magnetic field strength inside straight current carrying solenoid, directed along the axis of solenoid.

- 63. Refer to answer 62.
- 64. Refer to answer 62.
- **65.** (i) Magnetic field due to a current carrying solenoid,

 $B = \mu_0 nI$

where, n = number of turns per unit length

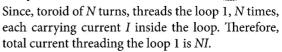
I =current flowing in the solenoid

$$B_N = B_2 - B_1$$

- $\Rightarrow \ B_N = \mu_0 n_2 I \mu_0 n_1 I \Rightarrow B_N = \mu_0 I (n_2 n_1).$
- (ii) Magnetic field at point outside the combined system is zero.
- **66.** (a) A solenoid bent into the form of closed loop is called toroid. The magnetic field *B* has a constant magnitude everywhere inside the toroid.
- (b) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.

Applying Ampere's circuital law,

$$\oint_{\text{loop }1} \vec{B} \cdot \vec{dl} = \mu_0(NI)$$



$$\Rightarrow \oint_{\text{loop }I} \vec{B} \cdot \vec{dl} = \mu_0(NI)$$

$$B \oint_{\text{loop }} dl = \mu_0(NI)$$

$$B \times 2\pi r = \mu_0 NI \text{ or } B = \frac{\mu_0 NI}{2\pi r}$$

(c) (i) Magnetic field inside the open space interior the toroid. Let the loop (2) as shown in figure experience magnetic field *B*.

No current threads the loop 2 which lies in the open space inside the toroid.

∴ Ampere's circuital law

$$\oint_{\text{loop } 2} \vec{B} \cdot \vec{dl} = \mu_0(0) = 0 \Longrightarrow B = 0$$

(ii) Magnetic field in the space exterior of toroid. Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

$$= NI - NI = 0$$

.. By Ampere's circuital law

$$\oint_{\text{loop }3} \vec{B} \cdot \overrightarrow{dl} = \mu_0(NI - NI) = 0 \Longrightarrow B = 0$$

Thus, there is no magnetic field in the open space interior and exterior the toroid.

67. (a) Ampere's circuital law states that the line integral of magnetic field induction \vec{B} around a closed path in vacuum is equal to μ_0 times the total current I passing through the surface, i.e., $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$



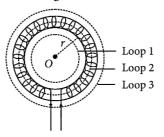
A toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. Consider an air-cored toroid with centre O. Given

r = Average radius of the toroid

I =Current through the solenoid

n = Number of turns per unit length

To determine the magnetic field inside the toroid, we consider three amperian loops (loop 1, loop 2 and loop 3) as shown in the figure:



For loop 1

According to Ampere's circuital law, we have

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \text{(Total current)}$$

Total current for loop 1 is zero because no current passes through this loop.

So, for loop 1

$$\oint \vec{B} \cdot \vec{dl} = 0$$

For loop 3

According to Ampere's circuital law, we have

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \text{(Total current)}$$

Total current for loop 3 is zero because net current coming out of this loop is equal to the net current going inside the loop.

For loop 2

The total current flowing through the toroid is NI, where N is the total number of turns.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0(NI) \qquad ...(i)$$

Now, \vec{B} and \vec{dl} are in the same direction

$$\oint \vec{B} \cdot \vec{dl} = B \oint dl$$

$$\Rightarrow \oint \vec{B} \cdot \vec{dl} = B(2\pi r) \qquad ...(ii)$$

Comparing (i) and (ii), we get

$$B(2\pi r) = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

Number of turns per unit length is given by

$$n = \frac{N}{2\pi r}$$

$$\therefore B = \mu_0 nI$$

This is the expression for magnetic field inside air-cored toroid.

(b) Given that the current flows in the clockwise direction for an observer on the left side of the solenoid. This means that left face of the solenoid acts as south pole and right face acts as north pole. Inside a bar magnet, the magnetic field lines are directed from south to north. Therefore, the magnetic field lines are directed from left to right in the solenoid.

Magnetic moment of single current loop is given by

$$m' = IA$$

where

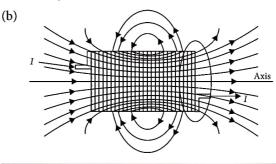
I =Current flowing through the loop

A =area of the loop

So, magnetic moment of the whole solenoid is given by

$$m = Nm' = N(IA)$$

68. (a) Refer to answer 62.



The magnetic field lines of magnet (or current carrying solenoid) form continuous closed loops and are directed from *N* to *S* pole outside the magnet and *S* to *N* pole inside the magnet and forms closed loops while in the case of an electric dipole, the field lines begin from positive charge and end on negative charge or escape to infinity.

69. A solenoid bent into the form of closed loop is called toroid. The magnetic field B has a constant magnitude everywhere inside the toroid.



Refer to answer 66(b, c).

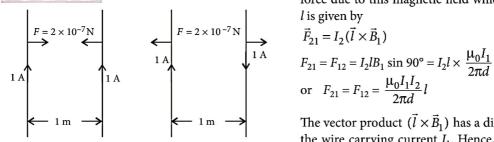
70. One ampere is the value of steady current which when maintained in each of the two very long, straight, parallel conductors of negligible cross section and placed one metre apart in vacuum, would produce on each of these conductors a force of attractive or repulsive nature of magnitude 2×10^{-7} N m⁻¹ on their unit length.

Force between two straight parallel current carrying

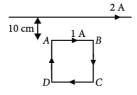
conductors,
$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r}$$
 when,

$$I_1 = I_2 = 1 \text{ A}, r = 1 \text{ m}, \text{ then}$$

 $F = 2 \times 10^{-7} \text{ N m}^{-1}$



71. Force between two parallel current carrying wires,



$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$$F_{AB} = \frac{\mu_0 \times 2 \times 1 \times 20 \times 10^{-2}}{2\pi \times 10 \times 10^{-2}}$$

 $= \frac{2\mu_0}{\pi} N \text{ (Attractive, towards the wire)}$

Force on arm CD;

$$F_{CD} = \frac{\mu_0 \times 2 \times 1 \times 20 \times 10^{-2}}{2\pi \times 30 \times 10^{-2}}$$

$$= \frac{2\mu_0}{3\pi} N$$
 (Repulsive, away from the wire)

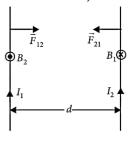
Force on arms BC and DA are equal and opposite. So, they cancel out each other.

Net force on the loop is $F = F_{AB} - F_{CD}$

$$= \frac{\mu_0}{\pi} \left[2 - \frac{2}{3} \right] = \frac{4\mu_0}{3\pi} = \frac{4 \times 4\pi \times 10^{-7}}{3\pi}$$

= 5.33×10^{-7} N (Attractive, towards the wire)

72. When two parallel infinite straight wires carrying currents I_1 and I_2 are placed at distance d from each other, then current I_1 produces magnetic field, which at any point on the second current carrying wire is



$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$
 directed inwards perpendicular to

So, this current (I_2) carrying wire then experiences a force due to this magnetic field which on its length

$$\begin{aligned} \vec{F}_{21} &= I_2 (\vec{l} \times \vec{B}_1) \\ F_{21} &= F_{12} = I_2 l B_1 \sin 90^\circ = I_2 l \times \frac{\mu_0 I_1}{2\pi d} \\ \text{or} \quad F_{21} &= F_{12} = \frac{\mu_0 I_1 I_2}{2\pi d} l \end{aligned}$$

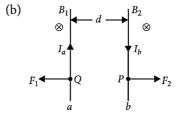
The vector product $(\vec{l} \times \vec{B}_1)$ has a direction towards the wire carrying current I_1 . Hence, both the wires attract each other.

So, force per unit length that each wire exerts on the

$$f = \frac{\mu_0 I_1 I_2}{2\pi d}$$
If $I_1 = I_2 = 1$ A and $d = 1$ m and $l = 1$ m
then $f = \frac{\mu_0}{2\pi} = 2 \times 10^{-7}$ N m⁻¹

Thus, electric current through each of two parallel long wires placed at distance of 1m from each other is said to be 1 ampere, if they exert a force of 2×10^{-7} N m⁻¹ on each other.

73. (a) Refer to answer 72.



Now, let the direction of current b be reversed. The magnetic field B_2 at point P due to current I_a flowing through a will be downwards. Similarly, the magnetic field B_1 at point Q due to current I_h passing through b will also, be downwards as shown. The force on a will be, therefore, towards the left. Also, the force on b will be towards the right. Hence, the two conductors will repel each other as shown.

74. The magnetic force of repulsion on the upper wire should be balancing its weight.

For wire CD to remain suspended at its position in equilibrium, magnetic force on CD due to AB =Weight of CD

$$\therefore \frac{\mu_0 I_1 I_2}{2\pi r} l = mg$$

$$m = 2 \times 10^{-7} \frac{I_1 I_2}{rg} \quad \text{(for } l = 1 \text{ m)}$$

$$= 1.2 \times 10^{-3} \text{ kg/m}$$

Current in CD should be in opposite direction to that in AB.

- 75. Refer to answer 72.
- 76. Refer to answer 72.
- 77. $I_1 = 2 \text{ A}$, $I_2 = 5 \text{ A}$, $a = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$ Force between two parallel wires per unit length is given by

$$F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{a}$$

= 2×10⁻⁷ × $\frac{2\times5}{1\times10^{-2}}$ = 20×10⁻⁵ N m⁻¹ (Repulsive)

78.
$$N = 200$$
, $B = 0.2$ T, $I = 5$ A Area, $A = 100$ cm² = 10^{-2} m², $\theta = 30^{\circ}$, $\tau = ?$

 $\tau = mB \sin \theta = NIAB \sin \theta$

$$= 200 \times 5 \times 10^{-2} \times 0.2 \times \sin 30^{\circ} = 1 \text{ Nm}$$

Coil will be in stable equilibrium if torque on it is zero *i.e.*, $\theta = O^0$.

It means plane of the coil should be perpendicular to the direction of magnetic field.

79. We have:

$$N_1 \cdot 2\pi R = N_2 \cdot 2\pi (R/2)$$

$$\therefore N_2 = 2N_1$$

Magnetic moment of a coil, M = NAI

For the coil of radius R

$$M_1 = N_1 I A_1 = N_1 I \pi R^2$$

For the coil of radius R/2

$$M_2 = N_2 I A_2 = 2N_1 I \pi R^2 / 4 = N_1 I \pi R^2 / 2$$

$$\Rightarrow M_2: M_1 = 1:2$$

80. (i) We know,
$$\vec{\tau} = \vec{M} \times \vec{B}$$

or
$$\tau = M B \sin \theta$$

$$0.063 = 0.9 \times B \times \sin 30^{\circ}$$

- or B = 0.14 T
- (ii) The position of minimum energy corresponds to position of stable equilibrium.

Energy
$$(U) = -MB \cos\theta$$

When $\theta = 0^{\circ} \Rightarrow U = -MB = \text{Minimum energy}$ Hence, when the bar magnet is placed parallel to the magnetic field, it is the state of stable equilibrium.

81. A revolving electron in an orbit of radius rmoving with velocity ν behaves as a current loop of effective current

$$I = ve$$
 (v is frequency of revolution)

$$=\frac{ve}{2\pi r}$$

Hence it acts like a magnetic dipole, thus

$$M = IA = \frac{ve}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

82. Here side of coil = 10 cm = 0.10 m,

$$n = 20$$
, $I = 12$ A, $\alpha = 30^{\circ}$, $\tau = 0.96$ Nm

Area of coil, $A = (\text{side})^2 = (0.1)^2 = 0.01 \text{ m}^2$

Torque $\tau = nIBA \sin \alpha$

$$0.96 = 20 \times 12 \times B \times 0.01 \times \frac{1}{2}$$

 $0.96 = 1.2 \times B \Rightarrow B = \frac{0.96}{1.2}$

$$0.96 = 1.2 \times B \implies B = \frac{0.96}{1.2}$$

- \therefore Magnetic field, B = 0.8 T
- **83.** Given N = 2000

$$A = 1.6 \times 10^{-4} \,\mathrm{m}^2$$

$$I = 4.0 \text{ A}$$

(i) Magnetic moment of solenoid,

$$m = NIA = 2000 \times 4.0 \times 1.6 \times 10^{-4} = 1.28 \text{ A m}^2$$

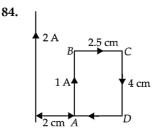
(ii) Torque $\tau = mB \sin\theta$

$$B = 7.5 \times 10^{-2} \,\mathrm{T}$$

$$\tau = 1.28 \times 7.5 \times 10^{-2} \times sin~30^{\circ}$$

$$= 1.28 \times 7.5 \times 10^{-2} \times 0.5$$

$$= 4.8 \times 10^{-2} \text{ N m}$$



Force between two current carrying wires,

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$
 Force on arm *AB*,

$$F_{AB} = \frac{\mu_0 \times 2 \times 1 \times 4 \times 10^{-2}}{2\pi \times 2 \times 10^{-2}}$$

 $= \frac{2\mu_0}{\pi} N$ (Attractive, towards the wire)

Force on arm CD,

$$F_{CD} = \frac{\mu_0 \times 2 \times 1 \times 4 \times 10^{-2}}{2\pi \times 4.5 \times 10^{-2}}$$
$$= \frac{8\mu_0}{9\pi} \text{ N (Repulsive, away from the wire)}$$

Force on arms BC and DA are equal and opposite. So, they cancel out each other.

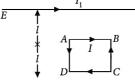
Net force on the loop is $F = F_{AB} - F_{CD}$

$$= \frac{\mu_0}{\pi} \left[2 - \frac{8}{9} \right] = \frac{10\mu_0}{9\pi} = \frac{10 \times 4\pi \times 10^{-7}}{9\pi}$$

= 4.44×10^{-7} N (Attractive, towards the wire)

85. The magnetic moment (\vec{m}) due to a planar square loop of side *l* carrying a steady current *I* is

 $\vec{m} = Il^2 \hat{n}$



The currents in AB and EF are flowing in the same direction. So AB will be attracted towards EF with a force, say F_1 .

$$\therefore F_1 = \frac{\mu_0}{2\pi} \frac{II_1}{l} \times \text{length of } AB$$

The currents in CD and EF are flowing in opposite directions. So CD would experience a repulsive force, say F_2 .

$$\therefore F_2 = \frac{\mu_0}{2\pi} \frac{II_1}{2l} \times \text{length of } CD$$

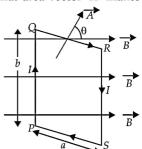
The forces on the portions BC and DA will cancel out each other's effect.

.. Net force =
$$F_1 - F_2$$

= $\frac{\mu_0}{2\pi} \cdot \frac{II_1}{l} \times \text{length of } AB - \frac{\mu_0}{2\pi} \cdot \frac{II_1}{2l} \times \text{length of } CD$
= $\frac{\mu_0}{2\pi} \cdot \frac{II_1}{l}$ [length of $AB - 1/2$ length of CD]

86. When a rectangular loop PQRS of sides 'a' and 'b' carrying current I is placed in uniform magnetic field \vec{B} , such that area vector \vec{A} makes

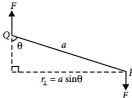
an angle θ with direction of magnetic field, then forces on the arms QR and SP of loop are equal, opposite and collinear, thereby perfectly cancel each other, whereas forces on the arms PQ and RS of loop are equal



and opposite but not collinear, so they give rise to torque on the loop.

Force on side PQ or RS of loop is

 $F = IbB \sin 90^{\circ} = Ib B$ perpendicular distance between two non-collinear forces is r_{\perp} $= a \sin \theta$



So, torque on the loop is

 $\tau = Fr_{\perp} = IbB \ a \sin \theta = I \ ba \ B \sin \theta$

or $\tau = IAB \sin \theta$

and if loop has N turns, then $\tau = NIAB \sin \theta$

In vector form, $\vec{\tau} = \vec{M} \times \vec{B}$

where $\vec{M} = N\vec{IA}$ is called magnetic dipole moment of current loop and is directed in direction of area vector \vec{A} i.e., normal to the plane of loop.

(a) If the plane of loop is normal to the direction of magnetic field i.e., $\theta = 0^{\circ}$ between \vec{B} and \vec{A} , then the loop does not experience any torque i.e.,

$$\tau_{\min} = 0$$

(b) If the plane of loop is parallel to the direction of magnetic field i.e., $\theta = 90^{\circ}$ between \vec{B} and \vec{A} , then the loop experiences maximum torque i.e.,

$$\tau_{\text{max}} = NIAB$$

- 87. Refer to answer 86.
- 88. Refer to answer 86.
- 89. When a current carrying coil is suspended in a uniform magnetic field, a torque acts on it, magnitude of which depends on the strength of current. This torque tends to rotate the coil about the axis of suspension, so that the magnetic flux passing through the coil is maximum.

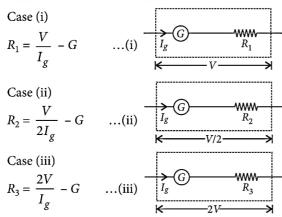
90. The moving coil galvanometers have low torsional constant (restoring torque per unit twist) to make it very sensitive.

91.
$$G = 30 \ \Omega$$
; $I_g = 2 \times 10^{-3} \text{A}$; $I = 0.3 \ \text{A}$

$$S = \frac{I_g G}{I - I_g} = \frac{(2 \times 10^{-3})30}{0.3 - 2 \times 10^{-3}} = 0.2 \ \Omega$$

92. Principle: A current carrying coil placed in a magnetic field experiences a torque, the magnitude of which depends on the strength of current.

Galvanometer as a voltmeter:



 I_g = current through galvanometer which is fixed. From eqns. (i) and (ii), we get

$$R_1 - R_2 = \frac{V}{2I_g} , G = R_1 - 2R_2$$

Put these values in eqn. (iii), we get

$$R_3 = 4(R_1 - R_2) - (R_1 - 2R_2) = 3R_1 - 2R_2$$

93. (a) Magnetic field is radial in moving coil galvanometer so that the plane of the coil always lies in the direction of the magnetic field. A radial magnetic field is produced by (i) properly cutting the magnetic pole pieces in the shape of concave faces and (ii) using a soft iron core within the coil.

(b)
$$V = (R + G)I_g$$
 ...(i)

$$V' = (R' + G)I_g \qquad \dots (ii)$$

From equations (i) and (ii)

$$\frac{V'}{V} = \frac{R' + G}{R + G} = \frac{1}{2} \implies R' = \frac{R}{2} - \frac{G}{2}.$$

94. (a) Current sensitivity: It is defined as the deflection of coil per unit current flowing in it, *i.e.*,

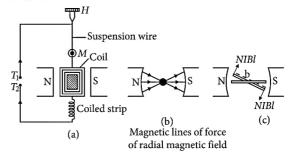
$$S = \frac{\Theta}{I} = \frac{NAB}{k}$$

(b) $A = 16 \times 10^{-4} \text{ m}^2$, N = 200, B = 0.2 T, $k = 10^{-6} \text{ Nm/degree}$, $\theta = 30^{\circ}$,

$$I = \frac{k}{NBA} \theta$$

$$= \frac{10^{-6} \times 30}{200 \times 0.2 \times 16 \times 10^{-4}} = 4.69 \times 10^{-4} \text{ A}.$$

95. A galvanometer is used to detect current in a circuit.



Principle and working: When current (*I*) is passed in the coil, torque τ acts on the coil, given by

$$\tau = NIAB \sin \theta$$

where θ is the angle between the normal to plane of coil and the magnetic field of strength B, N is the number of turns in a coil.

When the magnetic field is radial, as in the case of cylindrical pole pieces and soft iron core, then in every position of coil the plane of the coil, is parallel to the magnetic field lines, so that $\theta=90^{\circ}$ and $\sin 90^{\circ}=1$

Deflecting torque, $\tau = NIAB$

If C is the torsional rigidity of the wire and θ is the twist of suspension strip, then restoring torque = $C \theta$

For equilibrium, deflecting torque = restoring torque i.e. $NIAB = C \theta$

$$\therefore \quad \theta = \frac{NAB}{C}I \quad i.e., \ \theta \propto I$$

Deflection of coil is directly proportional to current flowing in the coil and hence we can construct a linear scale.

The uniform radial magnetic field keeps the plane of the coil always parallel to the direction of the magnetic field, *i.e.*, the angle between the plane of the coil and the magnetic field is zero for all the orientations of the coil.

96. Principle of working of a moving coil galvanometer: The working is based on the fact that

current carrying coil suspended in a magnetic field experiences a torque.

The galvanometer cannot be used to measure the value of the current in a given circuit due to following two reasons:

- (a) Galvanometer is a very sensitive device, it gives a full scale deflection for a current of the order of μA .
- (b) For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of the current in the circuit.

The current sensitivity of the galvanometer $\left(\frac{\phi}{I}\right)$ is given by

$$\frac{\Phi}{I} = \frac{NAB}{k}$$

- \therefore The current sensitivity of the galvanometer depends upon the number of turns N and cross section area of the coil A.
- 97. Let I be the total current and I_G should pass through the galvanometer and the remaining part $(I I_G)$ flows through the shunt S.

$$\therefore V_A - V_B = I_G G = (I - I_G)S$$

$$\therefore S = \left(\frac{I_G}{I - I_G}\right)G$$

$$\downarrow I_{-I_G}$$

$$\downarrow I_{-I_G}$$

$$\downarrow I_{-I_G}$$

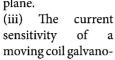
Here the range of an ammeter is 0 to 2I

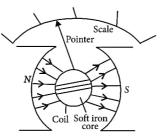
$$\therefore S' = \left(\frac{I_G}{2I - I_G}\right)G$$

98. (i) Refer to answer 95.

(ii) The cylindrical soft iron core, when placed inside the coil of a galvanometer, makes the magnetic field stronger and radial in the space between it

and pole pieces, such that whatever the position of the coil, the magnetic field is always parallel to its plane.





meter is defined as deflection of coil per unit current

passed through it. It is given by, $C.S. = \frac{NBA}{L}$

where N is the number of turns, A is the area of the coil, B is the magnetic field strength of the poles and k is the spring's constant of the suspension wire.

- 99. (a) Refer to answer 98.
- (b) Voltage sensitivity is given by, $V.S. = \frac{NBA}{kR}$ where *R* is the resistance of the wire.

Voltage sensitivity = Current sensitivity/R

Thus, on increasing the current sensitivity, voltage sensitivity may or may not increase because of similar changes in the resistance of the coil, which may also increase due to increase in temperature.

100. Refer to answers 98 and 99.

101. (a) Refer to answer 98.

- (b) A galvanometer can be converted into a voltmeter by connecting a high resistance in series with galvanometer to draw a very small current. A galvanometer can be converted into an ammeter by connecting a low resistance shunt in parallel with galvanometer to draw large value of current.
- **102.** Refer to answer 99.

103.
$$G = 50 \Omega$$

 $I_g = 5 \text{ mA} = 5 \times 10^{-3} \text{ A}$
 $V = 15 \text{ V}$

The galvanometer can be converted into a voltmeter when a high resistance R is connected in series with it.

Value of R is given by:

$$R = \frac{V}{I_g} - G = \frac{15}{5 \times 10^{-3}} - 50 = 2950 \ \Omega = 2.95 \ \text{k}\Omega$$

- 104. (a) and (b) Refer to answer 98.
- (c) Refer to answer 101(b).
- **105.** Voltage sensitivity is given by $\frac{\phi}{V} = \left(\frac{NAB}{k}\right) \frac{1}{R}$

Current sensitivity is given by $\frac{\phi}{I} = \left(\frac{NAB}{k}\right)$

According to the question

$$\frac{N'A'B}{k} = \frac{150}{100} \left(\frac{NAB}{k}\right),$$

$$N'A' = \frac{3}{2}NA, \ \frac{\phi'}{V} = \left(\frac{N'A'B}{k}\right)\frac{1}{R'}$$

$$\frac{\frac{3}{2}NAB}{k(2R)} = \frac{3}{4} \left(\frac{NAB}{k}\right) \frac{1}{R} = \frac{3}{4} \frac{\phi}{V}$$

Percentage decrease = $\frac{1 - \frac{3}{4}}{1} \times 100 = \frac{1}{4} \times 100 = 25\%$

106. (a) Refer to answer 95.

(b) Refer to answers 86 and 96.

